

Pensieve header: Searching for the EK-type equation for V.

```

SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.1"];
<< betaCalculus.m
Clear[ħ]; Unprotect[C];
$PerturbativeDegree = 8;
βSimplify[expr_] := Replace[
  Series[Normal[expr], {ħ, 0, $PerturbativeDegree}],
  sd_SeriesData => MapAt[Expand, sd, 3]
];
βCollect[B[ω_, μ_]] := B[
  βSimplify[ω],
  βSimplify[μ]
];
{V, C, sol} = Get["SolutionToDegree8-120524.m"];
{V, C} = {
  βCollect[
    B[ω[ħ c1, ħ c2], α[ħ c1, ħ c2] t[1] h[1] +
    β[ħ c1, ħ c2] t[1] h[2] + γ[ħ c1, ħ c2] t[2] h[1] + δ[ħ c1, ħ c2] t[2] h[2]]
  ] /. {
    (ε : (α | β | γ | δ | ω | κ)) [____] => ε0, (ε : (α | β | γ | δ | ω | κ)) (k-) [____] => εFromDigits[{k]}
  },
  βCollect[B[κ[ħ c1], 0]] /. {
    (ε : (α | β | γ | δ | ω | κ)) [____] => ε0,
    (ε : (α | β | γ | δ | ω | κ)) (k-) [____] => εFromDigits[{k]}
  }
} /. sol

```

$$\left\{ \begin{array}{l}
1 - \frac{1}{48} (c_1 c_2) \hbar^2 + \left(\frac{c_1^3 c_2}{2880} + \frac{17 c_1^2 c_2^2}{23040} + \frac{c_1 c_2^3}{2880} \right) \hbar^4 + \left(-\frac{c_1^5 c_2}{120960} - \frac{c_1^4 c_2^2}{35840} - \frac{103 c_1^3 c_2^3}{2580480} - \frac{c_1^2 c_2^4}{35840} - \frac{c_1 c_2^5}{120960} \right) \hbar^6 + \left(\frac{c_1^7 c_2}{4838400} \right) \hbar^8 \\
t[1] \\
t[2]
\end{array} \right.$$

```

{
  "R4" → R[2, 3] ** R[1, 3] ** V == V ** (R[1, 3] // dΔ[1, 1, 2]),
  "TwistEq" → V ** Θ[1, 2] == R[1, 2] ** (V // dP[2, 1]),
  "Unitarity" → V ** (V // dA[1] // dA[2]) == B[1, 0],
  "VerticalFlipEquation" → V ** (V // dS[1] // dS[2]) == R[1, 2],
  "CapEquation" → (V ** (C // dP[12]) // dcap[1] // dcap[2]) ==
    (C * (C // dP[2]) // dcap[1] // dcap[2]),
  "SidesNonDegeneracy" → (V // dη[1]) == B[1, 0] && (V // dη[2]) == B[1, 0],
  "CapsAndCups" → Simplify[C == (C // dS[1])],
  "OverhandEquation" →
    (Θ[V] // dΔ[1, 0, 1] // dS[2] // dS[3] // dm[0, 3, 0] // dm[1, 2, 1]) == B[1, 0]
}

```

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{R4 → True, TwistEq → True, Unitarity → True,
  VerticalFlipEquation → True, CapEquation → True, SidesNonDegeneracy → True,
  CapsAndCups → 2 c1 κ1 ħ +  $\frac{1}{48} c_1^3 \kappa_1 (-1 + 16 \kappa_1^2) \hbar^3 + \frac{c_1^5 \kappa_1 (13 - 160 \kappa_1^2 + 768 \kappa_1^4) \hbar^5}{46080} +$ 
 $(c_1^7 \kappa_1 (-153 + 1456 \kappa_1^2 - 5376 \kappa_1^4 + 12288 \kappa_1^6) \hbar^7) / 30965760 +$ 
  O[ħ]9 == 0, OverhandEquation → True}

```

C[[1]]

$$1 + c_1 \kappa_1 \hbar + \frac{1}{96} c_1^2 (-1 + 48 \kappa_1^2) \hbar^2 + \frac{1}{96} c_1^3 \kappa_1 (-1 + 16 \kappa_1^2) \hbar^3 + \frac{c_1^4 (13 - 480 \kappa_1^2 + 3840 \kappa_1^4) \hbar^4}{92160} +$$

$$\frac{c_1^5 \kappa_1 (13 - 160 \kappa_1^2 + 768 \kappa_1^4) \hbar^5}{92160} + \frac{c_1^6 (-51 + 1456 \kappa_1^2 - 8960 \kappa_1^4 + 28672 \kappa_1^6) \hbar^6}{20643840} +$$

$$\frac{1}{61931520} c_1^7 \kappa_1 (-153 + 1456 \kappa_1^2 - 5376 \kappa_1^4 + 12288 \kappa_1^6) \hbar^7 +$$

$$(c_1^8 (1169 - 29376 \kappa_1^2 + 139776 \kappa_1^4 - 344064 \kappa_1^6 + 589824 \kappa_1^8) \hbar^8) / 23781703680 + O[\hbar]^9$$

Log[C[[1]]] // Simplify

$$c_1 \kappa_1 \hbar - \frac{1}{96} c_1^2 \hbar^2 + \frac{c_1^4 \hbar^4}{11520} - \frac{c_1^6 \hbar^6}{725760} + \frac{c_1^8 \hbar^8}{38707200} + O[\hbar]^9$$

{V, C} = {V, C} /. κ₁ → 0;

Simplify[C == (C // dS[1])]

True

Simplify[C == (C // dA[1])]

True

C

$$\left(1 - \frac{1}{96} c_1^2 \hbar^2 + \frac{13 c_1^4 \hbar^4}{92160} - \frac{17 c_1^6 \hbar^6}{6881280} + \frac{167 c_1^8 \hbar^8}{3397386240} + O[\hbar]^9 \right)$$

Log[C[[1]]] /. c₁ → 2 // Simplify

$$-\frac{\hbar^2}{24} + \frac{\hbar^4}{720} - \frac{\hbar^6}{11340} + \frac{\hbar^8}{151200} + O[\hbar]^9$$

■ Cup equation

```
simplify[
  ((C // dS[1] // dP[12]) ** (V // Inverse)) // hη[1] // hη[2]) ==
  (C // dS[1]) (C // dS[1] // dP[2])
]
True
```

■ Top cap left punctured.

$$\text{ek1} = \mathbf{v} * (\mathbf{C} // \mathbf{dP}[3]) // \mathbf{t}\eta[1] // \mathbf{d}\mathbf{m}[2, 3, 2] // \mathbf{d}\mathbf{S}[2] // \mathbf{h}\mathbf{m}[1, 2, 2] // \mathbf{dP}[2 \rightarrow 0]$$

$$\left(1 - \frac{1}{96} c_0^2 \hbar^2 + \frac{13 c_0^4 \hbar^4}{92160} - \frac{17 c_0^6 \hbar^6}{6881280} + \frac{167 c_0^8 \hbar^8}{3397386240} + O[\hbar]^9 \right)$$

$$\mathbf{t}[0]$$

■ Right cup left punctured.

$$\text{ek2} = \mathbf{v} * (\mathbf{C} // \mathbf{dP}[3] // \mathbf{d}\mathbf{S}[3]) // \mathbf{d}\mathbf{m}[3, 2, 2] // \mathbf{h}\eta[2] // \mathbf{t}\eta[1] // \mathbf{d}\mathbf{m}[1, 2, 0]$$

$$\left(1 - \frac{1}{96} c_0^2 \hbar^2 + \frac{13 c_0^4 \hbar^4}{92160} - \frac{17 c_0^6 \hbar^6}{6881280} + \frac{167 c_0^8 \hbar^8}{3397386240} + O[\hbar]^9 \right)$$

$$\mathbf{t}[0]$$

■ Right cup top punctured.

$$\text{ek3} = \mathbf{v} * (\mathbf{C} // \mathbf{dP}[3] // \mathbf{d}\mathbf{S}[3]) // \mathbf{d}\mathbf{m}[3, 2, 2] // \mathbf{h}\eta[2] // \mathbf{d}\mathbf{S}[1] // \mathbf{d}\mathbf{m}[2, 1, 0]$$

$$\left(1 + \frac{1}{96} c_0^2 \hbar^2 - \frac{c_0^4 \hbar^4}{30720} + \frac{41 c_0^6 \hbar^6}{61931520} - \frac{473 c_0^8 \hbar^8}{39636172800} + O[\hbar]^9 \right)$$

$$\mathbf{t}[0]$$

■ Top cap right punctured.

$$\text{ek4} = \mathbf{v} * (\mathbf{C} // \mathbf{dP}[3]) // \mathbf{t}\eta[2] // \mathbf{d}\mathbf{m}[1, 3, 1] // \mathbf{d}\mathbf{S}[1] // \mathbf{d}\mathbf{m}[2, 1, 0]$$

$$\left(1 - \frac{1}{96} c_0^2 \hbar^2 + \frac{13 c_0^4 \hbar^4}{92160} - \frac{17 c_0^6 \hbar^6}{6881280} + \frac{167 c_0^8 \hbar^8}{3397386240} + O[\hbar]^9 \right)$$

$$\mathbf{t}[0] \quad -\frac{1}{2} + \frac{c_0 \hbar}{8} - \frac{1}{48} c_0^2 \hbar^2 + \frac{1}{384} c_0^3 \hbar^3 - \frac{c_0^4 \hbar^4}{3840} + \frac{c_0^5 \hbar^5}{46080} - \frac{c_0^6 \hbar^6}{645120}$$

$$\mathbf{h}[0]$$

■ Left cup right punctured.

$$\text{ek5} = \mathbf{v} * (\mathbf{C} // \mathbf{dP}[3] // \mathbf{d}\mathbf{S}[3]) // \mathbf{d}\mathbf{m}[3, 1, 1] // \mathbf{h}\eta[1] // \mathbf{t}\eta[2] // \mathbf{d}\mathbf{m}[2, 1, 0]$$

$$\left(1 - \frac{1}{96} c_0^2 \hbar^2 + \frac{13 c_0^4 \hbar^4}{92160} - \frac{17 c_0^6 \hbar^6}{6881280} + \frac{167 c_0^8 \hbar^8}{3397386240} + O[\hbar]^9 \right)$$

$$\mathbf{t}[0] \quad \frac{1}{2} + \frac{c_0 \hbar}{8} + \frac{1}{48} c_0^2 \hbar^2 + \frac{1}{384} c_0^3 \hbar^3 + \frac{c_0^4 \hbar^4}{3840} + \frac{c_0^5 \hbar^5}{46080} + \frac{c_0^6 \hbar^6}{645120}$$

$$\mathbf{h}[0]$$

■ Left cup top punctured.

$$\text{ek6} = \mathbf{v} * (\mathbf{C} // \mathbf{dP}[3] // \mathbf{d}\mathbf{S}[3]) // \mathbf{d}\mathbf{m}[3, 1, 1] // \mathbf{h}\eta[1] // \mathbf{d}\mathbf{S}[2] // \mathbf{d}\mathbf{m}[1, 2, 0]$$

$$\left(1 - \frac{c_0 \hbar}{2} + \frac{13}{96} c_0^2 \hbar^2 - \frac{5}{192} c_0^3 \hbar^3 + \frac{119 c_0^4 \hbar^4}{30720} - \frac{17 c_0^5 \hbar^5}{36864} + \frac{2813 c_0^6 \hbar^6}{61931520} - \frac{97 c_0^7 \hbar^7}{24772608} + \frac{12247 c_0^8 \hbar^8}{39636172800} + O[\hbar]^9 \right)$$

$$\mathbf{t}[0] \quad -\frac{1}{2} + \frac{c_0 \hbar}{8} - \frac{1}{48}$$

■ Identifications

$\text{ek2} * \text{ek3}$

$$(1 + O[\hbar]^9)$$

$(\text{ek1} * \text{ek1} // \text{Inverse})$

$$\left(1 + \frac{1}{48} c_0^2 \hbar^2 + \frac{c_0^4 \hbar^4}{23040} + \frac{c_0^6 \hbar^6}{1548288} - \frac{67 c_0^8 \hbar^8}{7431782400} + O[\hbar]^9 \right)$$

$$\mathbf{t}[0]$$

$$\text{Series}\left[\sqrt{\frac{\text{Sinh}[\hbar/2]}{\hbar/2}}, \{\hbar, 0, 8\}\right]$$

$$1 + \frac{\hbar^2}{48} + \frac{\hbar^4}{23\,040} + \frac{\hbar^6}{1\,548\,288} - \frac{67\hbar^8}{7\,431\,782\,400} + O[\hbar]^9$$

Log[ek5[[1]]]

$$-\frac{1}{96}c_0^2\hbar^2 + \frac{c_0^4\hbar^4}{11\,520} - \frac{c_0^6\hbar^6}{725\,760} + \frac{c_0^8\hbar^8}{38\,707\,200} + O[\hbar]^9$$

Log[ek6[[1]]]

$$-\frac{c_0\hbar}{2} + \frac{1}{96}c_0^2\hbar^2 - \frac{c_0^4\hbar^4}{11\,520} + \frac{c_0^6\hbar^6}{725\,760} - \frac{c_0^8\hbar^8}{38\,707\,200} + O[\hbar]^9$$